Prediction of seismic hazard

Prediction of seismic hazaed

- Time, location and intensity of earthquakes are probabilistic.

➔ Intensity and damage is probabilistic.

Probability evaluation of earthquake occurrence is necessary for prediction of damage to buildings

- Today's contents
  - Probabilistic model for occurrence of EQ
  - Analysis of seismic hazard

  Ref.: Akenori Shibata, "Analysis of Structural Safety by probabilistic approach"

Probabilistic model of EQ occurrence

- Annual probability *N(m)* for occurrence of EQ with magnitude *m* or more in target region.

$$N(m) = \frac{L(m)}{T}$$

T: duration

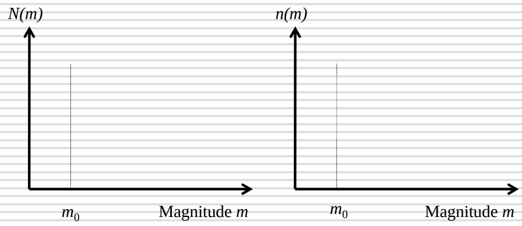
$L(m)$: number of EQs with magnitude *m* or more in T years

- Density function for average occurrence number

$$n(m) = -\frac{d}{dm}N(m)$$

Probabilistic model of EQ occurrence

- EQs with magnitude lower limit m_0, or more are taken into consideration.



Gutenberg-Richter Law (G-R eq.)

- Exponential function is suitable for *N*(*m*) of previous earthquake data.

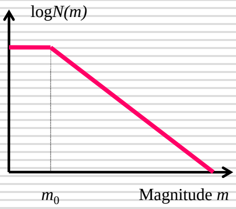
$$\log N(m) = a - bm \quad or \quad \ln N(m) = \alpha - \beta m$$

$$where, \alpha = a \ln 10, \ \beta = b \ln 10$$

G-R eq.

$$N(m) = e^{\alpha - \beta m} = N_0 e^{-\beta(m-m_0)}$$

$$n(m) = -\beta N_0 e^{-\beta(m-m_0)}$$



Probabilistic function of EQ magnitude

- probabilistic distribution function in a region $F_M(m)$

  ➔ probability of occurrence of EQ with magnitude of *m* or less

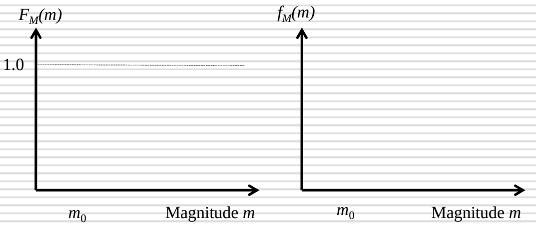
$$F_M(m) = \frac{N_0 - N(m)}{N_0} = 1 - \frac{N(m)}{N_0} = 1 - e^{-\beta(m-m_0)}$$

- probabilistic density function $f_M(m)$

  ➔ probability of occurrence of EQ with magnitude *m*

$$f_M(m) = \beta e^{-\beta(m-m_0)}$$

Probabilistic function of EQ magnitude



Example 1

- Activity of earthquakes in a region is represented by average annual occurrence shown in the table below.
- Evaluate probabilistic density function of EQ magnitude.
  - Consider magnitude m_0=6 or more.

Magnitude m	Average annual occurrence N(m)
6	0.1
7	0.01
8	0.001

Analysis of seismic hazard

Analysis of seismic hazard is…

- Prediction of probability of intensity of seismic motion at a location in a period.

Probabilistic distribution of maximum magnitude in *t* years.

(intensity of maximum seismic motion is more important for seismic design than number of earthquakes)

- Assumption 1: Imaginary point source is assumed.
- Assumption 2: G-R eq. is applied to relation of magnitude and number of occurrence.
- Assumption 3: Simple poisson process is applied to occurrence of earthquakes.

[Ref.] Simple Poisson Process

Probability of *k*-times occurrence in duration *t* *p(k,t)*

- Number of occurrence in unit time is ν.
- Divide duration *t* into *n* period with *k*-times occurrence and (n-k) times un-occurrence. $\Delta t = t/n$
- Probability of occurrence in Δt is $\nu\Delta t$.
- Probability *p(k,t)* can be given based on the binominal theorem.

$$p(k,t) = {}_nC_k(\nu\Delta t)^k(1-\nu\Delta t)^{n-k} = \frac{n!}{k!(n-k)!}(\nu\Delta t)^k(1-\nu\Delta t)^{n-k}$$

$$\text{when} \quad n=\infty, \quad p(k,t) = \frac{(\nu t)^k}{k!}e^{-\nu_0 t}$$

Analysis of seismic hazard

Probability of occurrence for *k*-times of EQs (over m_0) in t years, *p(k,t)*

$$p(k,t) = \frac{(N_0 t)^k}{k!}e^{-N_0 t}$$

Probabilistic distribution function of maximum magnitude in *t* years

$$F_{M\max}(m) = \sum_{k=0}^{\infty}\frac{(N_0 t)^k}{k!}e^{-N_0 t}\{F_M(m)\}^k = e^{-N_0 t}\sum_{k=0}^{\infty}\frac{\{N_0 t F_M(m)\}^k}{k!}$$

$$= e^{-N_0 t}\cdot e^{N_0 t F_M(m)} = e^{-N_0 t\{1-F_M(m)\}}$$

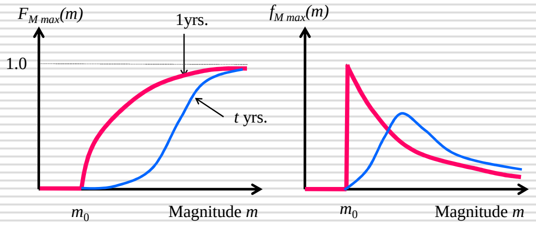
$$= e^{-N_0 t e^{-\beta(m-m_0)}}$$

Modified using characteristic value u_t

$$F_{M\max}(m) = e^{-e^{-\beta(m-u_t)}} \qquad u_t = m_0 + \frac{\ln(N_0 t)}{\beta}$$

Maximum magnitude m in t years

$$F_{M\max}(m) = e^{-e^{-\beta(m-u_t)}} \qquad f_{M\max}(m) = \beta e^{-\beta(m-u_t)-e^{-\beta(m-u_t)}}$$



Maximum ground motion in t years

- Seismic intensity, PGV and PGA are necessary for damage evaluation of buildings.
- Attenuation model with magnitude *m*, distance *r* and ground motion *y* is widely used.

Attenuation function

$$y = a_1 \cdot e^{a_2 m} \cdot r^{-a_3}$$

Where, $a_1,\ a_2,\ a_3$ are constants for of source and transfer characteristics.

Maximum ground motion in t years

$y = a_1 \cdot e^{a_2 m} \cdot r^{-a_3}$ is modified to

$$m = \frac{1}{a_2}(\ln y - \ln a_1 + a_3 \ln r)$$

Introduction of m into $F_{Mmax}(m)$ gives probability distribution function of maximum ground motion y.

$$F_{Y\max}(y) = e^{-N_0 t \cdot C \cdot r^{-\gamma} \cdot y^{-(\beta/a_2)}}$$

Where, $C = e^{\beta m_0} \cdot a_1^{\beta/a_2}$ $\gamma = \beta \frac{a_3}{a_2}$

Maximum ground motion in t years

Seismic ground y_0 for lower limit magnitude of m_0

$$y_0 = a_1 \cdot e^{a_2 m_0} \cdot r^{-a_3} = \left(C \cdot r^{-\gamma}\right)^{1/k} \qquad k = \frac{\beta}{a_2}$$

Probabilistic function of maximum ground motion in *t* years

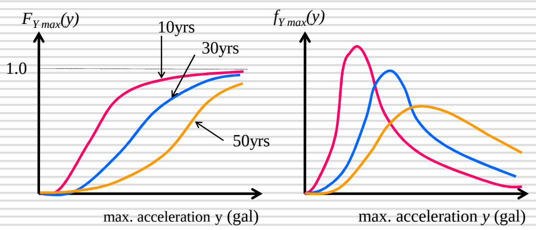
$$F_{Y\max}(y) = e^{-N_0 t \cdot (y_0/y)^k} = e^{-(v_t/y)^k}$$

$$f_{Y\max}(y) = \frac{k}{v_t} \cdot \left(\frac{v_t}{y}\right)^{k+1} \cdot e^{-(v_t/y)^k}$$

Where, $v_t = y_0 \cdot (N_0 t)^{1/k}$

Maximum ground motion in t years

$$F_{Y\max}(y) = e^{-N_0 t \cdot (y_0/y)^k} = e^{-(v_t/y)^k} \qquad f_{Y\max}(y) = \frac{k}{v_t} \cdot \left(\frac{v_t}{y}\right)^{k+1} \cdot e^{-(v_t/y)^k}$$



Seismic hazard curve (probability of exceedence)

Probability of ground motion of *y* or more in *t* years.

$$p_{yt} = 1 - F_{Y\max}(y) = 1 - e^{-(v_t/y)^k}$$