

NON LINEAR SEISMIC RESPONSE OF R/C FRAME STRUCTURE AND EQUIVALENT SINGLE-DEGREE-OF-FREEDOM SYSTEM

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ABSTRACT

Nonlinear seismic analyses were carried out for reinforced concrete frame structures which were designed on the basis of AIJ's "Design Guidelines for Earthquake Resistant Reinforced Concrete Buildings Based on Ultimate Strength Concept"[1]. The effect of hysteretic characteristics of beams on the response was studied. A fundamental oscillation mode was extracted from calculated horizontal displacement response. A whole frame was reduced to an equivalent single-degree-of-freedom system. It was shown possible to approximate seismic displacement response of the frame by the nonlinear analysis of an equivalent single-degree-of-freedom system.

1. INTRODUCTION

Reinforced concrete frame structures, expected to develop the yield mechanism of beam yield type during a strong earthquake motion, were designed in accordance with the AIJ's "Design Guidelines for Earthquake Resistant Reinforced Concrete Buildings Based on Ultimate Strength Concept" (Guidelines)[1]. The effect of hysteresis characteristics of beams on earthquake responses of frame structures was studied. A method to reduce a whole frame to an equivalent single-degree-of-freedom (SDOF) system was studied.

2. DESIGNED FRAME STRUCTURES

Four regular and symmetrical frame structures (6, 9, 12 and 15-stories high) were selected (Table 1). An analytical model of a frame, consisting of a column with its both side beams, was removed from the original structure by cutting beams at their inflection points and by supporting the beam ends with horizontal rollers (Fig. 1). Sections of members, arrangement of main reinforcement and strengths of concrete were listed in Table 2. Strengths and sizes of main reinforcement were shown in Table 3. Unit weight of frames was assumed to be 1.2 ton/m², sum of dead and live loads for earthquake loading. The foundation was assumed to be supported by firm sand gravel layer.

Table 1 List of Frames

Name	Story	Initial Period
A06	6	0.37sec
A09	9	0.55sec
A12	12	0.73sec
A15	15	0.91sec

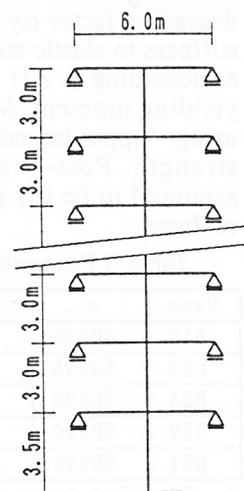


Fig. 1 Analytical Model

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Design for earthquake loading was based on AIJ's Guidelines[1].

Yield mechanism design : Yield hinges were planned at the ends of all the beams and at the bottom of the first story columns. Design earthquake loads was calculated by assuming standard base shear coefficient, C_B , to be equal to 0.25. Vibration characteristic factor, R_v , was calculated according to AIJ's Standard[2]. In linear structural analyses, stiffness was assumed to be 0.5 times as elastic stiffness for beams and 0.7 times for the first story columns. Dimensions of sections were selected so that story drift angle calculated under earthquake loads was not more than 1/300 rad. Reinforcement of all the beams and the first story columns intended to develop yield hinges were arranged for design stresses, which were attained by redistribution of the stresses obtained by linear analysis under design earthquake loads.

Yield mechanism assuring design : Yield mechanism assuring design was based on static nonlinear analysis assuming upper bound strength to develop at planned yield hinges. Frame analyzing program "DANDY"[3] was used in static nonlinear analysis. Lineal members were idealized by one-component model with nonlinear rotational springs at the ends of an elastic member. "Takeda model"[4] was used in rotational springs. Flexural cracking moment, M_c , and stiffness degrading factor α_y , the ration of yielding stiffness to elastic stiffness, were calculated according to AIJ Standard[2]. Flexural yielding moment, M_y , was calculated by using "upper bound strength" as material strength. Post-yielding stiffness was assumed to be 0.1 percent of the initial stiffness.

Table 3 Properties of Reinforcement

Name	σ_y	d_b	A_s	ϕ
D10	SD295	0.95	0.71	3.0
D22	SD295	2.22	3.87	7.0
D25	SD390	2.54	5.07	8.0
D29	SD390	2.86	6.42	9.0
D32	SD390	3.18	7.94	10.0

σ_y : Yielding strength(kg/cm²)

SD295: 3000(Reliable), 3900(Upper Bound)

SD390: 4000(Reliable), 5000(Upper Bound)

d_b : Diameter(cm), A_s : Area(cm²)

ϕ : Perimeter(cm)

Table2 Dimention of Members and Arrangement of Reinforcement (a) (A15)

Story	σ_B (kg/cm ²)	Column		Beam	
		B × D (cm)	Main bar	B × D (cm)	Main Bar Top Bottom
R	240			45x75	4-D22 4-D22
15	"	80x80	24-D25	"	6-D22 6-D22
14	"	"	"	50x80	5-D25 5-D25
13	300	85x85	24-D29	"	6-D25 6-D25
12	"	"	"	"	7-D25 7-D25
11	"	"	"	55x85	7-D29 6-D29
10	"	90x90	"	"	7-D29 7-D29
9	"	"	"	"	"
8	"	"	"	60x90	7-D32 6-D32
7	360	95x95	24-D32	"	"
6	"	"	"	"	7-D32 7-D32
5	"	"	"	"	"
4	"	"	"	"	"
3	"	"	"	"	"
2	"	"	"	"	"
1	Top	"	"	"	"
	Bottom	"	20-D29	"	"

Table2 (b) (A12)

Story	σ_B (kg/cm ²)	Column		Beam	
		B × D (cm)	Main bar	B × D (cm)	Main Bar Top Bottom
R	240			40x70	4-D22 3-D22
12	"	75x75	24-D22	"	6-D22 5-D22
11	"	"	"	45x75	5-D25 5-D25
10	"	80x80	"	"	6-D25 6-D25
9	"	"	24-D25	50x80	7-D25 7-D25
8	300	85x85	"	"	8-D25 7-D25
7	"	"	"	55x85	7-D29 7-D29
6	"	90x90	24-D29	"	"
5	"	"	"	"	8-D29 7-D29
4	"	"	"	60x85	8-D29 8-D29
3	"	"	"	"	"
2	"	"	"	"	8-D29 7-D29
1	Top	"	"	"	"
	Bottom	"	20-D29	"	"

Table2 (c) (A09)

Story	σ_B (kg/cm ²)	Column		Beam	
		B × D (cm)	Main Bar	B × D (cm)	Main Bar Top Bottom
R	240			40x70	3-D22 3-D22
9	"	75x75	24-D22	"	5-D22 5-D22
8	"	"	"	45x75	5-D25 4-D25
7	300	80x80	24-D25	"	6-D25 5-D25
6	"	"	"	"	7-D25 6-D25
5	"	"	"	50x80	6-D29 6-D29
4	"	85x85	24-D29	"	"
3	"	"	"	"	"
2	"	"	"	"	"
1	Top	"	"	"	"
	Bottom	"	20-D29	"	"

Table2 (d) (A06)

Story	σ_B (kg/cm ²)	Column		Beam	
		B × D (cm)	Main Bar	B × D (cm)	Main Bar Top Bottom
R	240			35x65	4-D22 4-D22
6	"	70x70	24-D22	"	6-D22 5-D22
5	"	"	"	"	6-D22 6-D22
4	"	75x75	24-D25	40x70	6-D25 5-D25
3	"	"	"	"	6-D25 6-D25
2	"	"	"	"	6-D25 5-D25
1	Top	"	"	"	"
	Bottom	"	20-D25	"	"

σ_B : Concrete Compressive Strength(kg/cm²)

Static nonlinear analysis was carried out under earthquake loads corresponding to an inverted triangular distribution of seismic coefficient. Design moment and shear for columns planned to develop no hinge were obtained by considering magnification due to dynamic effect and currency of bi-directional earthquake action. Reinforcement of columns was arranged for design moment and shear.

3. NONLINEAR SEISMIC ANALYSIS OF FRAME STRUCTURES

Nonlinear seismic analyses of the four frames (A06, A09, A12 and A15) were carried out to investigate the effect of hysteretic parameters of beams on earthquake response.

Earthquake records : Three earthquake accelograms were used ; NS components of the 1978 Tohoku Univ. record (TOH), NS components of the 1940 El Centro record (ELC) and EW components of the 1968 Hachinohe record (HAC). The three earthquake accelograms were magnified to maximum ground velocity of 80 cm/sec.

Table 4 Earthquake Accelograms

Model properties : The hysteretic model of columns was "Takeda Model"[4]. "Takeda model"[4] (Fig. 2) and "Takeda Slip Model"[5] (Fig. 3) were used as hysteretic model of beams to investigate the effect of hysteretic model on seismic response. Flexural cracking moment, M_c , and stiffness degrading factor, α_y , of members were calculated according to AIJ Standard[2]. Flexural yielding moment, M_y , was calculated by using "reliable strength" as material strength. Post-yielding stiffness was assumed to be 0.1 percent of the initial stiffness. Unloading stiffness degrading parameter, α , slipping stiffness degrading parameter, β , and slip hardening parameter, γ , was assumed to be 0.4, 1.2 and 1.1, respectively (Figs. 2 and 3). The initial damping factor were assumed to be 0.05; The damping was assumed to vary proportional to instantaneous stiffness.

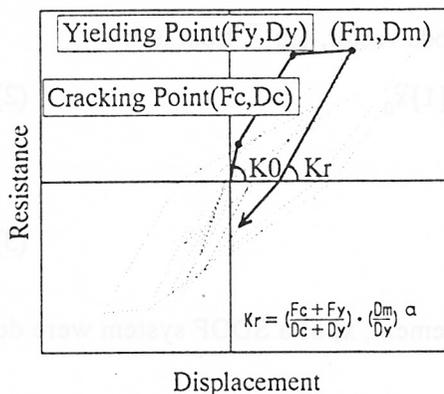


Fig.2 Takeda model

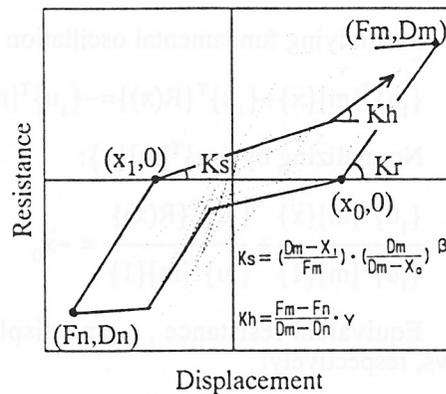


Fig.3 Takeda Slip Model

Seismic response : Maximum response ductility factors of beams were shown in Fig. 4. Maximum response ductility factors using Takeda slip model (S) were larger than those using Takeda model (T) for the same earthquake motion. However, the effect of different hysteresis models on the seismic response of frames was negligible. Maximum response ductility factor decreased in the order of Frames A06, A09, A12 and A15. Frames A06 and A09 developed yield mechanism for all the earthquake motions. However, beams of upper stories in frame A12 and A15 did not yield for ELC and HAC motions.

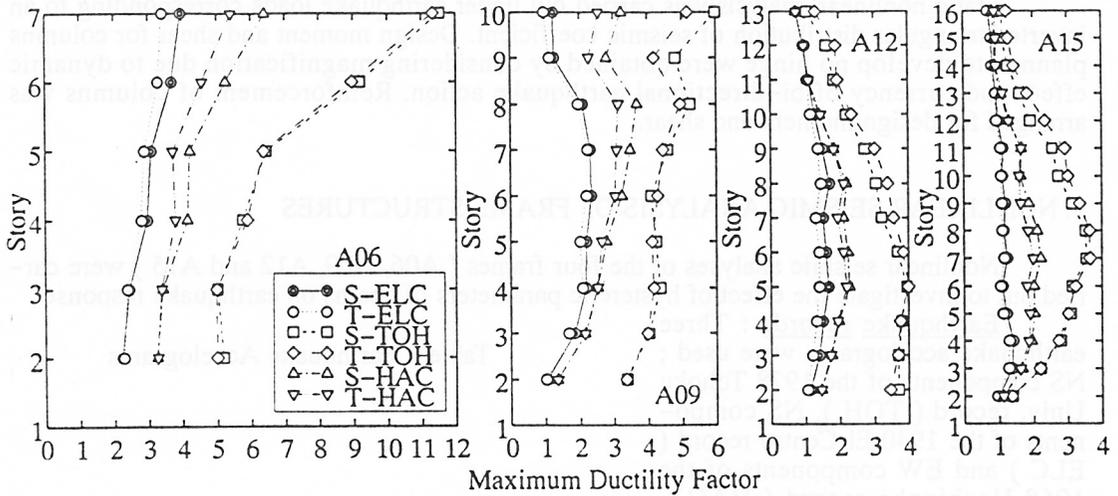


Fig. 4 Maximum Ductility Factors of Frames

4. EQUIVALENT SINGLE-DEGREE-OF-FREEDOM SYSTEM

Reduction to an equivalent single-degree-of-freedom system : Equation of motion of an undamped multi-degree-of-freedom system under horizontal ground motion was expressed as Eq.(1):

$$[m]\{\ddot{x}\} + \{R(x)\} = -[m]\{1\}\ddot{x}_0 \tag{1}$$

in which,

$[m]$ = mass matrix ; $\{x\}$ = vector of floor displacements relative to ground ; $\{R(x)\}$ = vector of floor forces ; $\{1\}$ = vector consisting of 1 ; \ddot{x}_0 = horizontal ground acceleration.

Multiplying fundamental oscillation mode vector, $\{1u\}^T$, from left:

$$\{1u\}^T[m]\{\ddot{x}\} + \{1u\}^T\{R(x)\} = -\{1u\}^T[m]\{1\}\ddot{x}_0 \tag{2}$$

Normalizing by $\{1u\}^T[m]\{1\}$:

$$\frac{\{1u\}^T[m]\{\ddot{x}\}}{\{1u\}^T[m]\{1\}} + \frac{\{1u\}^T\{R(x)\}}{\{1u\}^T[m]\{1\}} = -\ddot{x}_0 \tag{3}$$

Equivalent resistance , p , and displacement, x , of a SDOF system were defined as follows, respectively:

$$\bar{x} = \frac{\{1u\}^T[m]\{x\}}{\{1u\}^T[m]\{1\}}, \quad \bar{p} = \frac{\{1u\}^T\{R(x)\}}{\{1u\}^T[m]\{1\}} \tag{4}$$

The equation of motion of an equivalent SDOF system (mass = 1) was obtained as follows, by substitution of Eq.(4) into Eq.(3).

$$\ddot{\bar{x}} + \bar{p} = -\ddot{x}_0 \tag{5}$$

Fundamental oscillation mode : A fundamental oscillation mode, $\{1u\}$, continuously

changes with damages in a frame. A constant fundamental oscillation mode, however, was used to reduce a whole system to an equivalent SDOF in previous researches. A fundamental mode vector, $\{u\}$, was assumed to be proportional to a displacement distribution when yielding occurred or an inverted triangular distribution in Refs. 5 and 6, respectively. An initial fundamental mode was used in Ref. 7. Yoshimura et al.[8] and Takizawa[9] proposed a method to extract a time-averaged mode from time series of horizontal displacements obtained by seismic analysis.

In this paper, on the basis of Takizawa's research, a time-averaged mode was extracted from the horizontal displacement response. The fundamental mode vector $\{u\}$, extracted from vectorial time series of horizontal displacements, $\{x(t)\}$ ($t_0 < t < t_1$), was given by eigenvalue solution of Eq.(6).

$$([m][R][m])\{u\} = \lambda_1 [m]\{u\} \quad (6)$$

in which $[R] = \int_{t_0}^{t_1} \{x(t)\}\{x(t)\}^T dt.$

The extracted time-averaged modes (fundamental mode) of the four frames were shown in Fig. 5. The effects of hysteresis models of beams and earthquake motions on extracted fundamental modes were not remarkable.

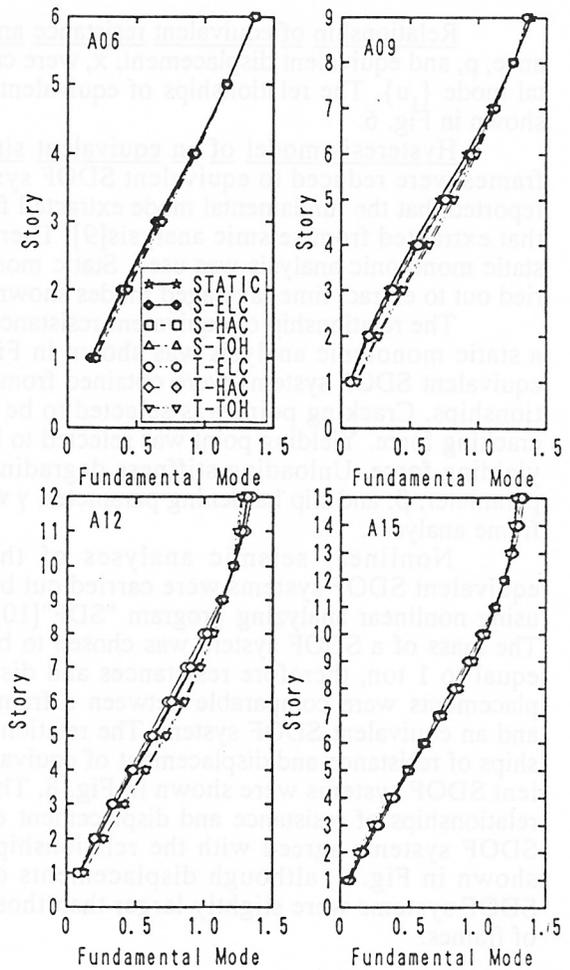


Fig. 5 Extracted Fundamental Modes

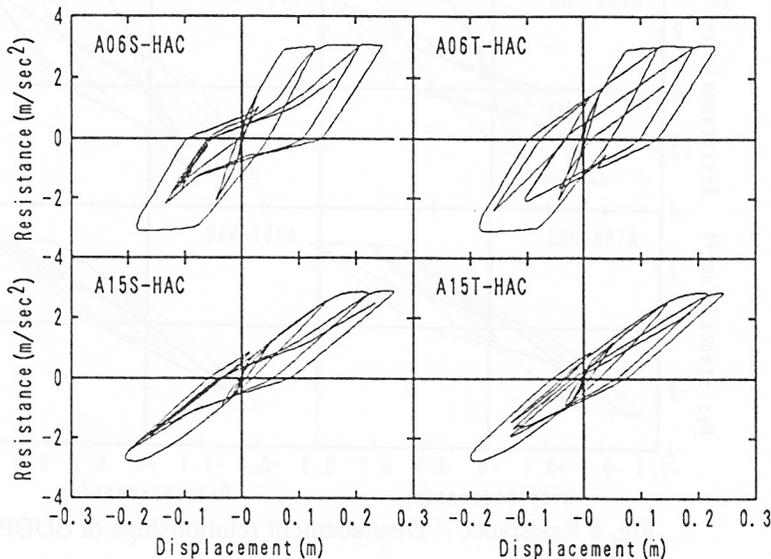


Fig. 6 Equivalent Resistance - Displacement Relationships of Frames

Relationship of equivalent resistance and equivalent displacement : Equivalent resistance, p , and equivalent displacement, x , were calculated by Eq.(4) using extracted fundamental mode $\{u\}$. The relationships of equivalent resistance and equivalent displacement were shown in Fig. 6.

Hysteresis model of an equivalent single-degree-of-freedom system : The four frames were reduced to equivalent SDOF systems to predict seismic response. Takizawa reported that the fundamental mode extracted from a static monotonic analysis was similar to that extracted from seismic analysis[9]. Therefore, the fundamental mode extracted from static monotonic analysis was used. Static monotonic analyses of the four frames were carried out to extract time-averaged modes shown in Fig.5 (STATIC).

The relationship of equivalent resistance and equivalent displacement calculated from a static monotonic analysis was shown in Fig. 7. Primary curves of hysteresis model of equivalent SDOF systems were obtained from equivalent resistance and displacement relationships. Cracking point was selected to be the point, at which one of members reached cracking force. Yielding point was selected to be the point, at which one of members reached yielding force. Unloading stiffness degrading parameter, α , slipping stiffness degrading parameter, β , and slip hardening parameter, γ were assumed to be the same values used in the frame analyses.

Nonlinear seismic analyses of the equivalent SDOF systems were carried out by using nonlinear analyzing program "SDF"[10]. The mass of a SDOF system was chosen to be equal to 1 ton, therefore resistances and displacements were comparable between a frame and an equivalent SDOF system. The relationships of resistance and displacement of equivalent SDOF systems were shown in Fig. 8. The relationships of resistance and displacement of SDOF systems agreed with the relationships shown in Fig. 6, although displacements of SDOF systems were slightly larger than those of frames.

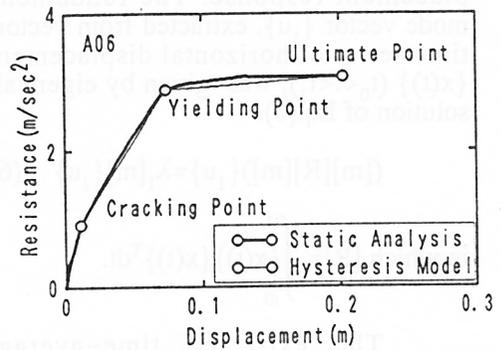


Fig. 7 Primary Curve of Hysteresis Model

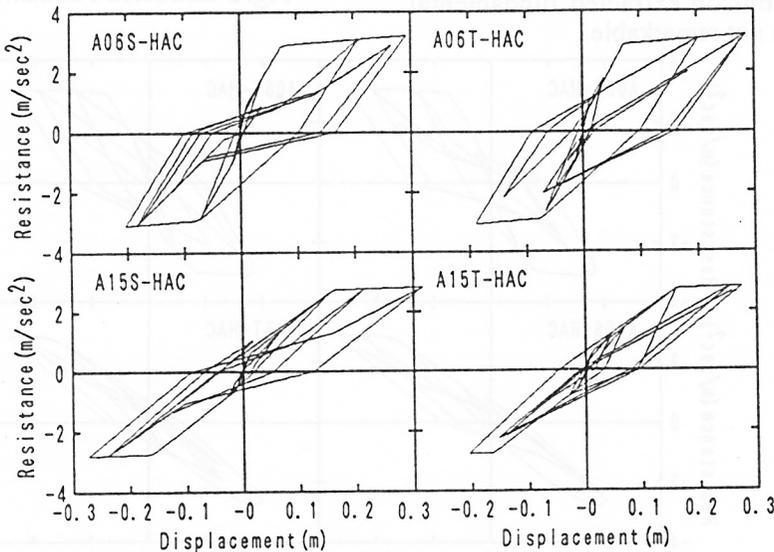


Fig. 8 Resistance - Displacement relationships of SDOF

Discussion :

Resistance response waveforms were compared in Fig. 9. The waveforms were very similar. Displacement response waveforms were compared in Fig. 10. The waveforms of a frame and an equivalent SDOF system were generally similar. There is a tendency that SDOF systems produce slightly larger displacements.

Comparison of maximum response displacements of frames and SDOF systems was shown in Fig. 11. Maximum response displacements of SDOF systems were 1.08 times, in average, as those of frames. Therefore, it is possible to approximate maximum displacement of a frame by nonlinear seismic analysis of an equivalent SDOF system described in this paper.

5. CONCLUSIONS

Conclusions are summarized as follows:

(1) Difference of hysteretic behavior of beams did not significantly influence the maximum ductility factor of beams and the seismic response of frames.

(2) It was possible to predict maximum response displacements of frame structures by seismic analysis of an equivalent single-degree-of-freedom.

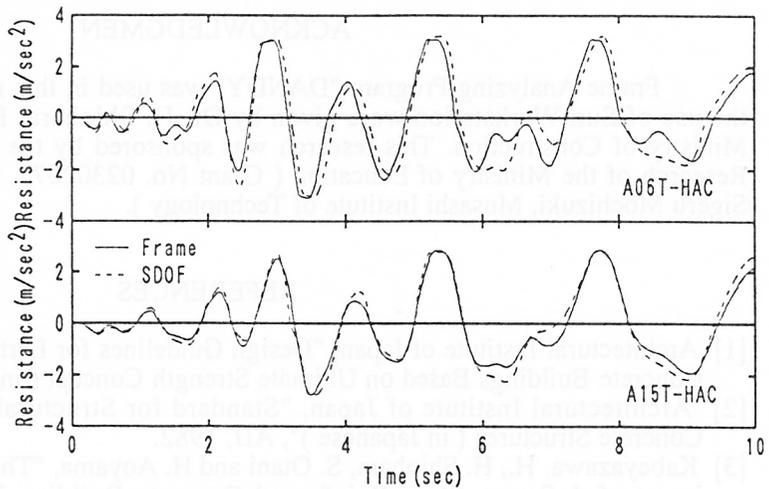


Fig. 9 Resistance Response Waveform

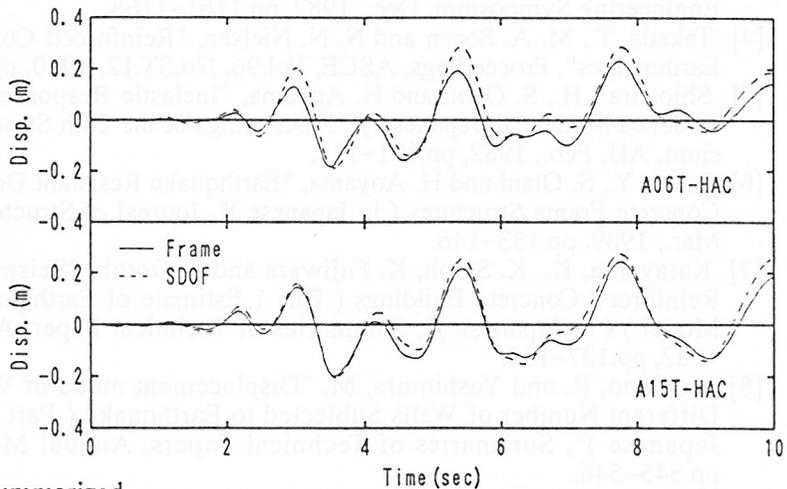


Fig. 10 Displacement Response Waveform

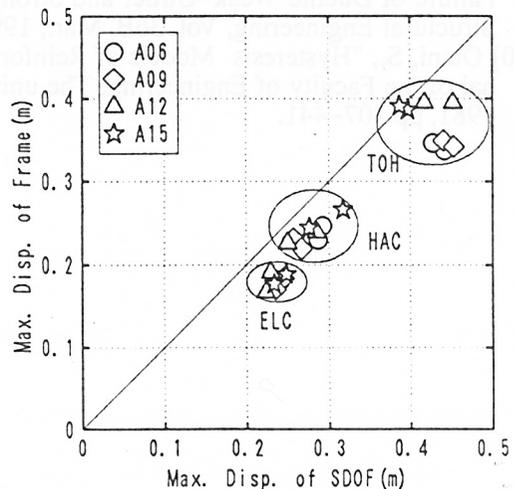


Fig. 11 Maximum Displacement of SDOF systems and Frames

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